

Competitive Energy Trading Framework for Demand-side Management in Neighborhood Area Networks

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Abstract—This paper, by comparing three potential energy trading systems, studies the feasibility of integrating a community energy storage (CES) device with consumer-owned photovoltaic (PV) systems for demand-side management of a residential neighborhood area network. We consider a fully-competitive CES operator in a non-cooperative Stackelberg game, a benevolent CES operator that has socially favorable regulations with competitive users, and a centralized cooperative CES operator that minimizes the total community energy cost. The former two game-theoretic systems consider that the CES operator first maximizes their revenue by setting a price signal and trading energy with the grid. Then the users with PV panels play a non-cooperative repeated game following the actions of the CES operator to trade energy with the CES device and the grid to minimize energy costs. The centralized CES operator cooperates with the users to minimize the total community energy cost without appropriate incentives. The non-cooperative Stackelberg game with the fully-competitive CES operator has a unique Stackelberg equilibrium at which the CES operator maximizes revenue and users obtain unique Pareto-optimal Nash equilibrium CES energy trading strategies. Extensive simulations show that the fully-competitive CES model is the most appropriate method for achieving effective demand-side management.

Index Terms—Community energy storage, demand-side management, game theory, neighborhood area network.

I. INTRODUCTION

Smart grid developments facilitate reliable and economical demand-side management for leveling peak energy demands and reducing energy costs [1]. Small-scale demand-side management as in residential gated communities has received attention with the increasing popularity and cost reductions of household-distributed renewable power generation and storage technologies. Community energy storage (CES) devices can be integrated with novel small-scale demand-side management approaches to efficiently utilize onsite energy generation from consumer-owned renewable power resources such as rooftop photovoltaic (PV) systems [2]. These approaches can create value for end users by reducing energy costs without modifying their electricity demand patterns [3].

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Small-scale demand-side management with a CES device and behind-the-meter PV systems requires a feasible framework with suitable incentives for all players. One common approach is to devise the load management optimization that is controlled by a centralized entity. However, robust operation of such a system would require system-wide information available to the controller including users' energy information that may increase the communication overhead [4]. In addition, the residential users may not subscribe to such a demand-side management approach as the central entity controls their personal energy decisions. A decentralized framework that can distribute the energy decision-making to individuals would be an effective alternative to overcome the above challenges.

In this paper, three energy trading systems with different CES operator structures are compared: a centralized cooperative CES operator that collaborates with users to minimize the total community energy costs, a benevolent CES operator that has socially favorable regulations with competitive consumers, and a fully-competitive CES operator in a non-cooperative Stackelberg game. The latter two systems use game-theoretic approaches where the CES operator (leader) moves first to maximize revenue. Users (followers) then follow the CES operator's actions to independently determine optimal CES energy trading strategies in a non-cooperative finitely repeated game. The fully-competitive CES operator has two degrees of freedom to maximize revenue in a non-cooperative Stackelberg game: energy price and their energy transactions with the grid. Conversely, the benevolent CES operator's ability to maximize revenue is restricted with only one degree of freedom, i.e., their energy transactions with the grid, in a Stackelberg game. We have the following main contributions in this work:

- The non-cooperative Stackelberg game between the fully-competitive CES operator and users has a unique Stackelberg equilibrium where users obtain unique Pareto-optimal Nash equilibrium CES energy trading strategies.
- Extensive numerical analysis demonstrates,
 - 1) The Stackelberg equilibrium of the fully-competitive system obtains a balance between social costs of the CES operator and energy users.
 - 2) Compared to the centralized cooperative system, both CES operator and participating users are simultaneously benefited in the fully-competitive system while the grid experiences load leveling.
 - 3) The community economic benefit is greater with

the fully-competitive system than the benevolent CES model, and the fully-competitive system can be implemented effectively with the least CES battery storage capacity of the three models.

The majority of Stackelberg game-theoretic demand-side management methods in literature exploit demand flexibility of consumers to obtain optimal system-wide objectives [5]–[7]. For example, the Stackelberg game in [6] achieves the optimal load control of electrical appliances through an effective real time pricing method. To the best of our knowledge, few game-theoretic works achieve optimal load management by utilizing energy from distributed energy resources as an alternative to reshaping consumer demand profiles [8], [9]. For example, the non-cooperative game in [9] determines optimal power settings of consumer-owned controllable power sources, such as gas turbines and energy storage devices, to minimize energy costs. In contrast to the use of consumer-owned energy storage devices as in [9], we study the use of increased flexibility of a centralized CES device to utilize uncontrollable and intermittent PV power generation to achieve demand-side management without reshaping user demand. To this end, we investigate the leader-follower interaction between the CES operator and the users using Stackelberg games.

This work has two key differences to [10] where a non-cooperative dynamic game between users is studied evaluating only users' autonomy to minimize costs. First, here, we devise bi-level energy trading systems to incorporate autonomies of both CES operator and users to minimize energy costs using Stackelberg games. We also investigate the tradeoff between the CES capacity and community benefits, whereas [10] does not impose energy capacity constraints for the CES device by assuming that it has sufficient capacity at all times.

The remainder of this paper is structured as follows. Section II presents related work. The system models are described in Section III, and Section IV describes the centralized energy trading system. The two game-theoretic energy trading systems are discussed in Section V. Section VI discusses simulation results, and conclusions are drawn in Section VII.

II. RELATED WORK

There is a rich literature on demand-side management that exploits user demand flexibility to achieve economic power system improvements. For example, dynamic pricing for consumption scheduling [11], load shifting methods [1], [12], [13], and incentive-based demand response programs [14], [15] have been investigated. We study demand-side management with a CES device to utilize household-distributed PV power generation without modifying users' energy demands.

Prior works have examined centralized control of distributed power resources, such as renewable power sources and storage devices, for effective energy management [16], [17]. Decentralized control of energy resources has been proposed to increase system reliability and robustness [4]. In particular, game theory has been applied to analyze independent interactions between distributed energy resources in power system [18], [19]. Authors in [20] achieve cost-effective energy management through a non-cooperative game that schedules consumer-owned energy storage devices and appliances.

Stackelberg game theory has been widely used to study interactions between electricity utilities (or electricity aggregators) and consumers for optimal demand-side management [21]–[23]. In the Stackelberg game in [24], utility determines the optimal energy price and users determine the optimal energy consumption schedules to minimize energy costs in response to the utility price signals. The Stackelberg game in [25] yields important insights for optimal demand-side management between utility and energy consumers by considering price-elasticity of consumer demand, and the Stackelberg game between users and a shared-facility controller in [26] improves the efficacy of demand-side management by managing consumer demand. Unlike prior work, the non-cooperative Stackelberg game here explores optimal energy trading coordination between a centrally-located CES device and residential PV energy users who also trade energy with the main power grid. Moreover, we focus on exploiting onsite generation from user-owned PV systems for demand-side management as an alternative to energy consumption scheduling.

III. SYSTEM CONFIGURATION

A. Demand-Side Model

The demand-side of the community is divided into participating users \mathcal{A} and non-participating users \mathcal{P} . The users \mathcal{A} have their own PV panels without local energy storage devices, and they participate in the energy management optimization by trading energy with the grid and/or the CES device. We assume that each user in \mathcal{A} has a decision-making controller in their household to perform their local energy trading optimization. The users \mathcal{P} consume energy only from the grid as they do not have local power generation capabilities and do not participate in the demand-side management optimization.

The users \mathcal{A} are subdivided into two time-dependent categories: surplus users $\mathcal{S}(t)$ and deficit users $\mathcal{D}(t)$. The users $\mathcal{S}(t)$ have surplus energy from their PV power generation after meeting energy demand, and the users $\mathcal{D}(t)$ have net energy deficits. We divide the time period of analysis \mathcal{T} , typically one day, into H equal time steps of length Δt with discrete time $t = 1, 2, \dots, H$. We consider $|\mathcal{S}(t)| = I_{\mathcal{S}(t)}$, $|\mathcal{D}(t)| = I_{\mathcal{D}(t)}$, and $|\mathcal{A}| = I = I_{\mathcal{S}(t)} + I_{\mathcal{D}(t)}$.

At each time t , user $i \in \mathcal{S}(t)$ evaluates the optimal energy amount that they can sell to the CES device, and user $j \in \mathcal{D}(t)$ decides optimal energy amount that can be bought from the CES device. These strategies are determined day-ahead, and we assume that the users \mathcal{A} have accurate forecasts of their energy demands and PV power generation for the next day. For $n \in \mathcal{A}$, let $x_n(t)$, $l_n(t)$, $e_n(t)$, and $g_n(t)$ are the energy traded with the CES device, energy consumption from the grid, personal energy demand, and PV energy generation at time t , respectively. According to the energy balance

$$l_n(t) = x_n(t) + e_n(t) - g_n(t). \quad (1)$$

Note that $x_n(t)$ is positive when the user is charging (or selling energy to) the CES device and negative when discharging (or buying energy from) the CES device. The surplus energy of $n \in \mathcal{A}$ at time t , $s_n(t)$, is given by

$$s_n(t) = g_n(t) - e_n(t). \quad (2)$$

We also let

$$\begin{aligned} 0 &\leq x_i(t) \leq s_i(t), \quad \forall i \in \mathcal{S}(t), \quad t \in \mathcal{T}, \\ s_j(t) &\leq x_j(t) \leq 0, \quad \forall j \in \mathcal{D}(t), \quad t \in \mathcal{T}. \end{aligned} \quad (3)$$

B. Community Energy Storage Model

In this paper, the energy storage model is similar to that in [9]. At each time t , the CES device may exchange energy $l_Q(t)$ with the grid in addition to its energy transactions with the users \mathcal{A} . Here, $l_Q(t) > 0$ if the CES device is charging from the grid, and $l_Q(t) < 0$ if it is selling energy to the grid.

Without loss of generality, consider splitting the CES energy transaction vectors into separate positive charging and discharging vectors such that

$$x_n \equiv x_n^+ - x_n^-, \quad l_Q \equiv l_Q^+ - l_Q^-, \quad x_n^+, x_n^-, l_Q^+, l_Q^- \succeq \mathbf{0} \quad (4)$$

where \succeq indicates element-wise inequality, and $x_n, l_Q, x_n^+, x_n^-, l_Q^+, l_Q^-$ are H -dimensional column vectors with elements $x_n(t), l_Q(t), x_n^+(t), x_n^-(t), l_Q^+(t), l_Q^-(t)$, respectively, $\forall t \in \mathcal{T}$. Note that once inefficiencies are introduced, all optimal solutions satisfy $x_n^+(t)x_n^-(t) = 0$ and $l_Q^+(t)l_Q^-(t) = 0$ at all times to avoid simultaneous charging and discharging of the CES device [9]. We introduce charging and discharging inefficiencies $0 < \beta^+ \leq 1$ and $\beta^- \geq 1$, respectively, to consider conversion losses of the CES device. For instance, if x^+ energy is sold to the CES device, then the charge level only increases by $\beta^+ x^+$. Similarly, $\beta^- x^-$ energy must be discharged to obtain x^- energy from the CES device. If $q(t-1)$ is the charge level at the beginning of time t , then the charge level at the end of time t , $q(t)$, is given by

$$q(t) = \alpha q(t-1) + \beta^+ \chi^+ - \beta^- \chi^- \quad (5)$$

where α ($0 < \alpha \leq 1$) is the leakage rate, $\chi^+ = \left[\sum_{n=1}^I x_n^+(t) + l_Q^+(t) \right]$, and $\chi^- = \left[\sum_{n=1}^I x_n^-(t) + l_Q^-(t) \right]$.

Using (5), we write (6) to ensure the CES charge level within its energy capacity limit at each time t as

$$\mathbf{0} \prec q(0)\boldsymbol{\eta} + \Psi(\chi^+ - \chi^-)\boldsymbol{\beta} \preceq \mathbf{Q}_M \quad (6)$$

where $q(0)$ is the initial charge level, and $\mathbf{Q}_M \in \mathbb{R}^{H \times 1}$ with all its entries being Q_M that is the maximum energy capacity of the CES device. Additionally, $\boldsymbol{\eta} \in \mathbb{R}^{H \times 1}$ has $\eta(\tau) = \alpha^\tau$, $\Psi \in \mathbb{R}^{H \times H}$ is a lower triangular matrix that has elements $[\Psi]_{v,w} = \alpha^{v-w}$, $\boldsymbol{\beta} = [\beta^+, \beta^-]^T$, $\mathbf{0}$ is the H -dimensional zero column vector, and χ^+, χ^- are the H -dimensional column vectors with all their entries being χ^+, χ^- , respectively.

Assuming that the initial charge level $q(0)$ is within the CES device's safe operating region, we set (7) to ensure the continuous operation of the CES device for the next day and to prevent over-charging or over-discharging during \mathcal{T} [17]

$$q(H) = q(0). \quad (7)$$

C. Energy Cost Models

The unit electricity price of the grid at time t is assumed to have a constant baseline component and a variable real-time component that is proportional to the total grid energy load at time t [1], [9]. In this work, the total grid energy load

at time t is $L(t) = \sum_{n=1}^I l_n(t) + l_Q(t) + l_P(t)$ where $l_P(t)$ is the aggregate grid energy load of the users \mathcal{P} . Assuming $0 < L(t) < L_{\max}$ for non-negative pricing and L_{\max} is the overload condition of the grid, the unit electricity price of the grid at time t , $p(t)$, is given by

$$p(t) = \delta_t + \phi_t L(t) \quad (8)$$

where δ_t and ϕ_t are positive time-of-use tariff constants at time t that are determined according to a day-ahead market clearing process [9]. With similar analysis to [1], the resulting energy cost function of the grid at time t , $p(t)L(t)$, is a strictly convex function with respect to the total grid load $L(t)$.

In our game-theoretic systems, the CES operator adopts prices for energy transactions with the users \mathcal{A} . Then, the total energy cost of user $n \in \mathcal{A}$ at time t , $C_n(t)$, is given by

$$C_n(t) = p(t)l_n(t) - a(t)x_n(t) \quad (9)$$

where $a(t)$ is the unit price of the CES device at time t . Both fully-competitive and benevolent CES operators obtain revenue through energy trading with the grid and the users \mathcal{A} . Assuming that the CES operator exchanges energy with the grid at the grid energy price, we consider the CES revenue as

$$R = \sum_{t=1}^H \left(-a(t) \sum_{n=1}^I x_n(t) - p(t)l_Q(t) \right). \quad (10)$$

However, in the centralized energy trading system, the CES operator does not obtain revenue from energy trading with the users \mathcal{A} . In this regard, we do not consider a separate revenue function as (10), and the cost of user $n \in \mathcal{A}$ is derived as in (9) disregarding the term $a(t)x_n(t)$.

IV. CENTRALIZED ENERGY TRADING SYSTEM

This section describes the community energy trading system with a centralized cooperative CES operator that solves the optimization problem of minimizing the total energy cost paid by the entire community to the grid. Here, we assume that the users \mathcal{A} communicate their energy demand and PV energy generation profiles to the CES operator, and the operator also has the perfect knowledge of the community participation percentage. The CES operator schedules the energy transactions across the community by solving the optimization problem

$$\min \sum_{t=1}^H p(t)L(t) \quad (11)$$

subjects to constraints (3), (6), and (7).

Note that in this system, the CES operator has no price signal for their energy transactions with the users \mathcal{A} and consequently, has no direct incentive. It also requires impractical information exchange and cooperation. The cooperating participating users similarly do not have direct incentives as their personal energy costs may inflate for the benefit of the overall community. All of these reasons make the practical implementation of the centralized cooperative energy trading system less feasible.

V. DECENTRALIZED ENERGY TRADING SYSTEMS

In our decentralized energy trading systems, the CES operator interacts with the users \mathcal{A} to maximize their revenue (10) while each user $n \in \mathcal{A}$ minimizes their individual energy cost in (9) by manipulating $x_n(t)$. The CES operator's revenue and the energy costs of the users \mathcal{A} are coupled through individual's energy decisions $x_n(t)$, $a(t)$, and $l_Q(t)$. Thus, we develop Stackelberg game-theoretic frameworks to analyze the hierarchical CES-user energy trading interactions.

A. Objective of the Participating Users

Here, each user $n \in \mathcal{A}$ seeks to minimize their personal energy costs. Therefore, in response to any suitable H -dimensional energy price vector \mathbf{a} and grid load vector \mathbf{l}_Q of the CES operator, user $n \in \mathcal{A}$ minimizes their energy cost in (9) at each time $t \in \mathcal{T}$. The cost function (9) is quadratic with respect to both $l_n(t)$ and $x_n(t)$. We consider

$$C_n(t) = K_2 l_n(t)^2 + K_1 l_n(t) + K_0 \quad (12)$$

where $K_2 = \phi_t$, $K_1 = (\phi_t L_{-n}(t) + \delta_t - a(t))$, and $K_0 = -a(t)s_n(t)$. Here, $L_{-n}(t)$ is the total community load on the grid excluding the load of user n and $L_{-n}(t) = L(t) - l_n(t)$.

Since (12) depends on the actions of the other users $n' \in \mathcal{A} \setminus n$, we formulate a non-cooperative game $\Gamma \equiv \langle \mathcal{A}, \mathcal{X}, \mathcal{C} \rangle$ between the users \mathcal{A} at each time $t \in \mathcal{T}$ to determine their optimal strategies. Here, $\mathcal{X} = \mathbf{X}_1(t) \times \dots \times \mathbf{X}_I(t)$ where $\mathbf{X}_n(t)$ is the strategy set of user $n \in \mathcal{A}$ subject to constraints (3), and $\mathcal{C} = (C_1(t), \dots, C_I(t))$ is the set of cost functions of the users \mathcal{A} at time t . We denote $\mathbf{x}(t) = [x_1(t), \dots, x_I(t)]$. Each user $n \in \mathcal{A}$ selects their strategy $x_n(t) \in \mathbf{X}_n(t)$ to minimize the cost function $C_n(x_n(t), \mathbf{x}_{-n}(t)) \equiv C_n(t)$. Here, $\mathbf{x}_{-n}(t)$ is the CES energy transaction strategy profile of the users $n' \in \mathcal{A} \setminus n$. Therefore, each user $n \in \mathcal{A}$ determines

$$x_n(t) = \underset{x_n(t) \in \mathbf{X}_n(t)}{\operatorname{argmin}} C_n(x_n(t), \mathbf{x}_{-n}(t)). \quad (13)$$

To make the game-theoretic analysis tractable, we assume that the users \mathcal{A} have accurate day-ahead predictions of PV power generation and energy demand. Consequently, playing the game Γ at each time $t = 1, 2, \dots, H$ by the users \mathcal{A} using \mathbf{a} and \mathbf{l}_Q turns into a non-cooperative finitely repeated game with perfect information where Γ is the stage game [27].

Proposition 1. *For any given values of $a(t)$ and $l_Q(t)$, the stage game Γ obtains a unique pure-strategy Nash equilibrium.*

Proof: Nash equilibrium implies no player can gain by unilaterally changing their own strategy while the others play their Nash equilibrium strategies [27]. For feasible $\mathbf{x}_{-n}(t)$, (12) is strictly convex since its second derivative with respect to $x_n(t)$ is positive as $\phi_t > 0$ [28]. Therefore, each participating user's objective function in (13) is strictly convex. Additionally, the individual strategy sets are compact and convex due to linear inequalities (3). Therefore, a unique Nash equilibrium with pure strategies for the game Γ is obtained [29]. ■

For $n \in \mathcal{A}$, the Nash equilibrium response can be found using

$$\left. \frac{\partial C_n(t)}{\partial x_n(t)} \right|_{x_n(t)=\tilde{x}_n(t)} = 2K_2(\tilde{x}_n(t) - s_n(t)) + K_1 = 0. \quad (14)$$

By solving (14) for all participating users I , using the expressions of K_1 and K_2 in (12), the Nash equilibrium solution can be written as a function of the CES operator's output variables $a(t)$ and $l_Q(t)$

$$\begin{aligned} \tilde{x}_n(t) &= s_n(t) - \varepsilon(t), \\ \varepsilon(t) &= -(I+1)^{-1}[\phi_t^{-1}(a(t) - \delta_t) - l_P(t) - l_Q(t)]. \end{aligned} \quad (15)$$

Note that parameters δ_t and ϕ_t should be adequate such that $\tilde{x}_n(t)$ satisfies (3) for given $a(t)$, $l_Q(t)$, and $l_P(t)$.

B. Objective of the Community Energy Storage Operator

In the decentralized energy trading setting, the CES operator's primary objective is to maximize the revenue in (10). Therefore, if we substitute (15) into (10), the CES operator's utility maximization can be simplified to a quadratic optimization problem to determine optimal variables \mathbf{a} and \mathbf{l}_Q

$$[\mathbf{a}, \mathbf{l}_Q] = \underset{\mathbf{a}, \mathbf{l}_Q \in \mathcal{Q}}{\operatorname{argmax}} \sum_{t=1}^H (\lambda a(t)^2 + \mu a(t) + \nu l_Q(t)^2 + \xi l_Q(t)) \quad (16)$$

where \mathcal{Q} is the strategy set available to the CES operator, $\lambda = -I(I+1)^{-1}\phi_t^{-1}$, $\mu = I(I+1)^{-1}(l_P(t) + \phi_t^{-1}\delta_t) - \sum_{n=1}^I s_n(t)$, $\nu = -\phi_t(I+1)^{-1}$, and $\xi = -(I+1)^{-1}(\phi_t l_P(t) + \delta_t)$. The strategy set \mathcal{Q} is convex as it is only subject to linear constraints (6) and (7). The objective function in (16) is strictly concave since its Hessian matrix is negative definite for all $\mathbf{a}, \mathbf{l}_Q \in \mathcal{Q}$ as coefficients $\lambda, \nu < 0$. Therefore, (16) always has a unique maximum [28].

C. Benevolent CES Operator Model

After describing the objectives of the users \mathcal{A} and the CES operator, in this section, we analyze the Stackelberg energy trading competition between the benevolent CES operator and the users \mathcal{A} . Consider if $x_n(t) = s_n(t)$, $\forall n \in \mathcal{A}$ and $\forall t \in \mathcal{T}$, then these user strategies are at the Nash equilibrium of the game Γ if and only if $\varepsilon(t) = 0$ in (15). As a result

$$a(t) = \delta_t + \phi_t(l_Q(t) + l_P(t)). \quad (17)$$

In this model, we consider the price constraint (17) as an auxiliary constraint for the CES operator when maximizing their revenue in addition to (6) and (7). The objective function in (16) can be written as a function of $l_Q(t)$ only by substituting (17) into (16). Thus, the objective of the CES operator in the benevolent scenario can be given by

$$[l_Q] = \underset{l_Q \in \mathcal{Q}}{\operatorname{argmax}} \sum_{t=1}^H (\gamma_1 l_Q(t)^2 + \gamma_2 l_Q(t) + \gamma_3) \quad (18)$$

where $\gamma_1 = -\phi_t$, $\gamma_2 = -\delta_t - \phi_t(l_P(t) + \sum_{n=1}^I s_n(t))$, and $\gamma_3 = -\sum_{n=1}^I s_n(t)(\delta_t + \phi_t l_P(t))$.

In the Stackelberg competition, the CES operator, as the leader, firstly determines optimal $l_Q(t)$ by solving (18) and

then their energy price $a(t)$ using (17) for each time $t \in \mathcal{T}$. Using these values, the users \mathcal{A} determine optimal strategies of $x_n(t)$ by playing the game Γ at each time t . Note that in this setup, we restrict the CES operator's ability to set a price such that $\varepsilon(t) = 0$ in (15) by imposing the constraint (17). As a result, at the Nash equilibrium of the game Γ , the energy requirements of the users \mathcal{A} are shifted on to the CES device such that $\tilde{x}_n(t) = s_n(t)$. Hence, $\tilde{l}_n(t) = 0, \forall n \in \mathcal{A}, \forall t \in \mathcal{T}$.

In this system, the CES operator does not have full freedom to maximize the revenue in (16) with the additional constraint (17). Here, $\tilde{p}(t) = \delta_t + \phi_t(l_P(t) + \tilde{l}_Q(t)) = \tilde{a}(t)$ where $\tilde{l}_Q(t)$ is the CES device's grid load at its maximum revenue in (18) since the users \mathcal{P} and the CES device are the only remaining energy loads on the grid as the users \mathcal{A} shift their net energy requirements $s_n(t)$ to the CES device. We consider this as a benevolent CES operator that is regulated by the users \mathcal{A} to amalgamate all their energy requirements into one entity with storage capabilities for better demand-side management.

D. Fully-competitive CES Operator Model

This section explains the non-cooperative Stackelberg game between the fully-competitive CES operator and the users \mathcal{A} . In the system, the CES operator first sets a and l_Q to maximize their revenue in (16) and broadcasts them to the users \mathcal{A} . Using these signals, the users \mathcal{A} repeat the non-cooperative game Γ at each time t . Unlike the benevolent CES model, this system does not impose any price constraint for the CES operator. Hence, the CES operator's objective is identical to (16).

In this scenario, the bi-level interaction between the CES operator and the users \mathcal{A} can be formulated as a non-cooperative Stackelberg game Υ . We represent the strategic form of Υ as $\Upsilon \equiv \{\{\mathcal{L}, \mathcal{A}\}, \{\mathcal{Q}, \mathcal{X}\}, \{R, \mathcal{C}\}\}$ where the CES operator \mathcal{L} is the leader, the users \mathcal{A} are the followers, and all other notations are defined as in the preceding sections.

Proposition 2. *The game Υ obtains a unique Stackelberg equilibrium.*

Proof: The non-cooperative game Γ between the users \mathcal{A} has a unique Nash equilibrium for given $a(t)$ and $l_Q(t)$ of the CES operator (see Proposition 1). There is also a unique solution of the CES operator's revenue maximization (16). Therefore, the game Υ obtains a unique Stackelberg equilibrium as soon as the CES operator determines their unique revenue maximizing strategy $[a^*, l_Q^*]$ while the users \mathcal{A} play their unique Nash equilibrium strategy profile $x^*(t)$ at each time t . ■

The Stackelberg equilibrium strategies of the CES operator and the users \mathcal{A} , $\rho^* \equiv [a^*, l_Q^*]$ and $x^*(t)$, respectively, satisfy

$$C_n(x^*(t), \rho^*) \leq C_n(x_n(t), x_{-n}^*(t), \rho^*), \quad \forall n \in \mathcal{A}, \forall x_n(t) \in \mathbf{X}_n(t), \forall t \in \mathcal{T}, \quad (19)$$

$$R(\mathbf{X}^*, \rho^*) \geq R(\mathbf{X}^*, \rho), \quad \forall \rho \in \mathcal{Q} \quad (20)$$

where $x_{-n}^*(t)$ is the Nash equilibrium strategy profile of the users \mathcal{A} except user n at time t , and $\mathbf{X}^* = (x^*(1)^T, \dots, x^*(H)^T)$ is the H -tuple of Nash equilibrium strategy profiles of the users \mathcal{A} at each t in response to ρ^* .

Proposition 3. *At the Stackelberg equilibrium of the game Υ , the Nash equilibrium CES energy trading strategies of the users \mathcal{A} achieved by playing the game Γ are Pareto optimal.*

Proof: Let us consider the energy trading strategies of the users \mathcal{A} and the CES operator at the Stackelberg equilibrium of the game Υ at any $t \in \mathcal{T}$: $[x^*(t), a^*(t), l_Q^*(t)]$. Assume there is any feasible $x'(t) (\neq x^*(t))$ such that $x'(t)$ Pareto dominates $x^*(t)$. Define the sum of the CES energy trading amounts of the users \mathcal{A} at $x'(t)$ and $x^*(t)$ are $X'_A(t)$ and $X_A^*(t)$, respectively, where $X'_A(t) = X_A^*(t) + \theta X_A^*(t)$ and θ is a non-zero scalar. Due to the introduced change of $X_A^*(t)$ to $X'_A(t)$, the CES device is forced to over-charge or over-discharge violating (7). Therefore, the CES operator has to divert from their strategy $l_Q^*(t)$ to $l'_Q(t) = l_Q^*(t) - \theta X_A^*(t)$ in order to satisfy (7) that ensures the sustainable operation of the CES device throughout the day. For example, if the CES charge level rises due to the introduced change, the CES operator has to discharge the increased amount of energy to the grid. Note that, in doing so, the Stackelberg equilibrium grid price $p^*(t)$ does not change as the total grid load does not change.

Then at the new operating point, the CES operator's cost $C_{CES}(t)$ at time t can be given by

$$C_{CES}(t) = a^*(t)X'_A(t) + p^*(t)l'_Q(t), \quad (21)$$

and the sum of the costs of the users \mathcal{A} , $C_A(t)$, is

$$C_A(t) = p^*(t) \left(X'_A(t) - S_A(t) \right) - a^*(t)X'_A(t) \quad (22)$$

where $S_A(t) = \sum_{n=1}^I s_n(t)$. By substituting $X'_A(t) = X_A^*(t) + \theta X_A^*(t)$ and $l'_Q(t) = l_Q^*(t) - \theta X_A^*(t)$ into (21) and (22), it is evident that if $C_A(t)$ decreases, then $C_{CES}(t)$ increases and vice versa. Such a situation is not led by the CES operator and hence there is no feasible $x'(t) \in \mathcal{X} \setminus x^*(t)$ that Pareto dominates $x^*(t)$ after obtaining the Stackelberg equilibrium. In other words, for a given $[a^*(t), l_Q^*(t)]$, it is infeasible to adopt any $x'(t) \in \mathcal{X} \setminus x^*(t)$ such that $C_n(x'(t)) \leq C_n(x^*(t)), \forall n \in \mathcal{A}$ and $C_n(x'(t)) < C_n(x^*(t))$ for some $n \in \mathcal{A}$. This concludes the proof of the proposition. ■

A two-step iterative algorithm was used to determine the Stackelberg equilibrium in the game Υ as shown in Algorithm 1. In the first step, the CES operator determines a and l_Q , and in the second step, the users \mathcal{A} play the non-cooperative game Γ at each time t .

VI. RESULTS AND DISCUSSION

For numerical simulations, we consider a residential community of 40 people with 30%, 40%, and 50% participating users. We obtain the average daily domestic PV power generation and user electricity demand profiles from [30]. We use $H = 48$, $\Delta t = 30$ min, $Q_M = 80$ kWh, $q(0) = 0.25Q_M$, $\alpha = 0.9^{1/48}$, $\beta^+ = 0.9$, and $\beta^- = 1.1$ [9]. Parameter ϕ_t is selected in each case such that $\phi_{\text{peak}} = 1.5 \times \phi_{\text{off peak}}$ where the peak period is 16:00-23:00. The value of ϕ_{peak} is then set such that the predicted daily unit price range of the grid is the same as a reference time-of-use unit electricity price range used in Sydney, Australia [31]. δ_t is a constant across time such that

Algorithm 1 Game to obtain the Stackelberg equilibrium**Step 1:**

- 1: $r \leftarrow 1$ (r is an iteration number).
- 2: **if** $r = 1$
- 3: The CES operator initializes $[a, l_Q]$ and broadcasts to the users \mathcal{A} .
- 4: **else**
- 5: The CES operator solves (16) using X^* and broadcasts $[a, l_Q]$ to the users \mathcal{A} .
- 6: **end if**

Step 2:

- 7: $t \leftarrow 1$.
- 8: Initialize $x(t)$ such that $x_n(t) \in X_n(t)$; $\forall n \in \mathcal{A}$ and $k \leftarrow 0$ (k is an iteration number).
- 9: **repeat**
- 10: $k \leftarrow k + 1$.
- 11: **for** each user $n \in \mathcal{A}$ **do**
- 12: User n solves (13) using $[a, l_Q]$ and the aggregate load of the other users $n' \in \mathcal{A} \setminus n$ at the iteration $k - 1$.
- 13: **end for**
- 14: **until** there is no change in $x(t)$ at the iterations k and $k - 1$.
- 15: $t \leftarrow t + 1$ and repeat from 8 until $t = H$.
- 16: Announce X^* to the CES operator and $r \leftarrow r + 1$.
- 17: Repeat from 2 until there is no change in X^* at the iterations r and $r - 1$.
- 18: Return X^* and $[a^*, l_Q^*]$ as the Stackelberg equilibrium.

predicted average grid price matches the average price of the reference signal. To compare results, we consider a baseline energy trading system without a CES device where the users \mathcal{A} trade energy exclusively with the main power grid that has the same energy cost model in Section III. In particular, PV energy producers sell all surplus PV energy directly to the grid.

A. Preliminary Study of Three Energy Trading Systems

Consider the case with 40% participating users to demonstrate our CES models. In Fig. 1, we illustrate the variations of price signals of the CES operator and the grid in the fully-competitive CES model and the grid price of the baseline. Fig. 1 shows that the introduction of the CES device reduces the peak grid electricity price in the competitive CES model compared to the baseline. Before 09:00, when there is little PV energy and all participating users are deficit users, the CES operator sets a price above the equilibrium grid price such that it is unfavorable for any of the deficit users to purchase energy. Subsequently, the users \mathcal{A} may not buy energy from the CES device during this period. During the day, when PV energy is plentiful, and through the evening peak, when electricity demand is greatest, it is favorable for the CES operator to trade energy with the users \mathcal{A} . Therefore, at these times the CES price approaches the equilibrium grid price in a similar way to the benevolent CES model. In turn, the users \mathcal{A} transfer the majority of their energy transactions to the CES device. In the benevolent and the centralized cooperative CES models, there

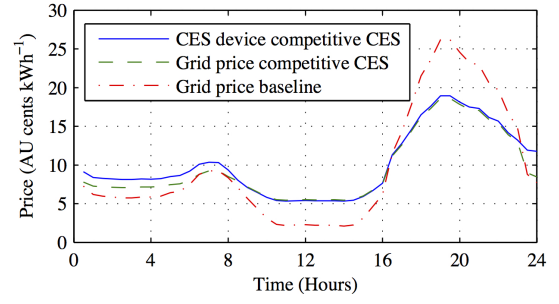


Fig. 1. Variation of electricity prices with 40% participating users in the fully-competitive CES model and the baseline.

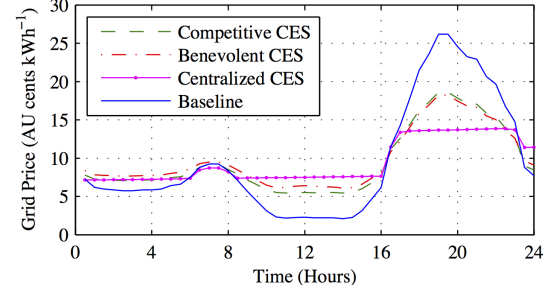


Fig. 2. Variation of grid electricity prices of the different CES operator models with 40% participating users.

are more energy transactions between the CES device and the users \mathcal{A} than in the fully-competitive CES model. Fig. 2 shows that this causes a greater reduction in the peak grid price.

More energy transactions between the CES device and the users \mathcal{A} require greater CES storage capabilities (see Fig. 3), and subsequently, there are more energy transactions between the CES device and the grid: the fully-competitive case requires 58% less absolute energy traded between the CES device and the grid than the benevolent case.

Fig. 4 shows the community benefits of the three systems. The community benefit is the sum of absolute electricity cost savings of the users $\mathcal{A} \cup \mathcal{P}$ compared to the baseline and the CES revenue. As part of the CES revenue obtains from energy costs incurred by the users \mathcal{A} (see (10)), the community benefit reflects the total reduction in costs paid by the community (all users and the CES operator) to the grid compared to the baseline. All models have optimal energy storage requirements corresponding to the peaks in Fig. 4. The fully-competitive CES model requires a battery capacity less than 70 kWh to provide peak community benefit compared to the other models. The centralized CES model considers load management of the entire community, not only the users \mathcal{A} , and therefore, requires a significantly larger storage capacity for optimal performance.

Table I compares the performance of the three systems over several metrics with different percentages of the users \mathcal{A} . Here, the percentage cost savings and the peak-to-average ratio reductions are calculated compared to the baseline. When combined with the CES inefficiencies and price signal limitations, the CES operator revenue reduces from the fully-competitive through benevolent to centralized case in each user-percentage case. Conversely, as the CES device enacts

TABLE I
PERFORMANCE OF THE THREE CES MODELS WITH DIFFERENT FRACTIONS OF PARTICIPATING USERS (PU). PAR IS PEAK-TO-AVERAGE RATIO.

Performance Metric	Average PU Cost Savings (%)			CES Operator Revenue (AU cents)			Community Benefit (AU cents)			PAR Reduction (%)		
PU Fraction	30%	40%	50%	30%	40%	50%	30%	40%	50%	30%	40%	50%
Competitive CES	27.6	29.4	31.4	323	344	373	945	1023	1123	30.3	31.7	33.1
Benevolent CES	30.7	32.4	34.9	229	191	160	856	852	889	33.8	35.9	38.3
Centralized CES	61.2	62.0	64.2	-22	-72	-150	1193	1267	1369	37.0	38.2	39.5

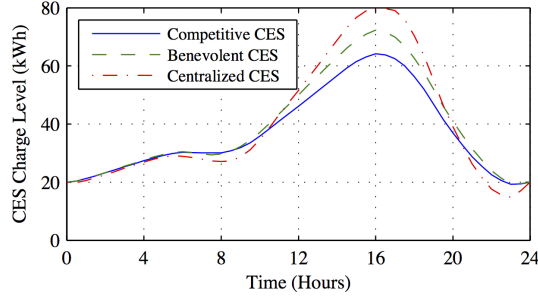


Fig. 3. Charge levels of the CES device for the different CES operator models with 40% participating users.

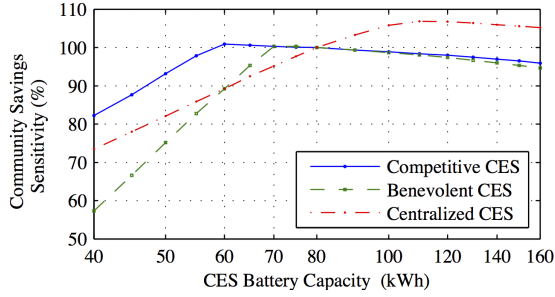


Fig. 4. Sensitivity of community benefit to energy storage capacity for the different CES operator models with 40% participating users.

greater demand-side management by reducing the peak-to-average ratio, the average cost saving of a user in \mathcal{A} increases. The average cost saving of a user in \mathcal{A} is similar under the fully-competitive and benevolent CES models, then increases notably under the centralized CES model. In fairness point of view, the users \mathcal{A} enjoy greater savings than the users \mathcal{P} . For example, with 40% participating users in the fully-competitive CES model, a participating user receives 29.4% cost saving on average and a non-participating user only receives 7.92% cost saving on average. The CES revenue is greatest for the fully-competitive CES model and decreases significantly to a loss under the centralized CES model (see Table I). This is because the fully-competitive model allows complete freedom to the CES operator to maximize revenue (16) while the benevolent CES operator is restricted to set a price and the centralized model eliminates the CES price signal. Overall, the total community benefit of introducing the CES device is greatest for the centralized CES followed by the fully-competitive CES and then the benevolent CES (see Table I). On average, the fully-competitive CES model provides 81% of the centralized model's economic benefit compared with only 68% for the benevolent CES model. Hence, the fully-

competitive CES model is the most effective and feasible model of the three systems.

Having insights for the feasibility of the fully-competitive CES model, to investigate the effects of imperfect energy forecasts on the system, we introduce proportional variance white noise errors to the PV power and energy demand forecasts [10]. When averaged over a large number of simulations, the mean absolute percentage energy forecast error is equal to half of percentage white noise variance. For 40% participating user case, when the mean absolute percentage forecast error changes from 0% to 50%, the average community user cost saving compared to the baseline was only reduced from 11.9% to 11.8%, and the participating user saving was reduced from 29.4% to 29.2% on average. Similar trends were observed for both 30% and 50% participating user cases. Therefore, the cost benefits of the fully-competitive CES model are robust to imperfect demand and PV energy forecasts.

B. Fully-competitive system versus socially-centralized system

Instead of implementing the decentralized fully-competitive CES model, it is possible to devise a socially-centralized system that minimizes the total social cost (sum of social costs of the users \mathcal{A} and the CES operator i.e., $\sum_{t=1}^H p(t)(\sum_{n=1}^I l_n(t) + l_Q(t))$ with constraints (3), (6) and (7). Such a system would provide an upper bound for comparing performance of our decentralized system. However, this centralized method requires private energy information of the users \mathcal{A} , such as their energy demands, available to the central controller. Furthermore, it does not incorporate a price signal for the CES operator similar to the centralized CES model in Section IV. In this regard, Table II compares social costs between the socially-centralized approach and the fully-competitive CES model for different fractions of the users \mathcal{A} . Compared to the fully-competitive CES model, the socially-centralized system generates an increasing loss for the CES operator with increasing fractions of the users \mathcal{A} despite the lower social costs for the users \mathcal{A} . This shows that the unique Stackelberg equilibrium of the hierarchical fully-competitive system assures the balance between social costs of the CES operator and the users \mathcal{A} as they play optimal best responses to each other. On average, the total social cost of the decentralized system is only 20.3% higher than that of the socially-centralized system. When the number of users \mathcal{A} increases from 30% to 50%, the increase of the total social cost of the decentralized system becomes less than 20% of the socially-centralized system.

TABLE II

COMPARISON OF SOCIAL COSTS BETWEEN THE DECENTRALIZED FULLY-COMPETITIVE CES MODEL AND THE SOCIALLY-CENTRALIZED ENERGY TRADING SYSTEM. PARTICIPATING USERS (PU).

Performance Metric	Social Cost of PU (AU cents)			Social Cost of CES Operator (AU cents)			Total Social Cost (AU cents)		
	30%	40%	50%	30%	40%	50%	30%	40%	50%
Competitive CES model	870	1180	1498	-323	-344	-373	547	836	1125
Socially-centralized System	400	605	727	51	88	219	451	693	946

VII. CONCLUSION

Community energy storage (CES) devices offer significant opportunities for user electricity cost savings, operator revenue, and peak-to-average ratio reduction of the grid. These benefits were shown to increase with the fraction of the participating users in the community. We have investigated three different CES operator models for community-level demand-side management and presented a fully-competitive CES model in a non-cooperative Stackelberg game that is the most cost-feasible structure as it delivers greater economic benefits with less energy storage capacity.

Interesting future work could focus on introducing more energy trading flexibility from different distributed generation sources and investigating fairness aspects between participation and non-participation in the decentralized fully-competitive energy trading system.

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